Three Logistic Models for the Two-Species Interactions: Symbiosis, Predator-Prey and Competition

R. López-Ruiz[†] and D. Fournier-Prunaret[‡]

[†] DIIS-BIFI, Universidad de Zaragoza,
Campus San Francisco, 50009 - Zaragoza (Spain).
[‡] SYD-LESIA, INSA Toulouse,
Campus Rangueil, 31077 - Toulouse Cedex (France).

Abstract

If one isolated species is supposed to evolve following the logistic mapping, then we are tempted to think that the dynamics of two species can be expressed by a coupled system of two discrete logistic equations. As three basic relationships between two species are present in Nature, namely symbiosis, predator-prey and competition, three different models are obtained. Each model is a cubic two-dimensional discrete logistic-type equation with its own dynamical properties: stationarity, periodicity, quasi-periodicity and chaos. Furthermore, these models could be considered as the basic ingredients to construct more complex interactions in the ecological networks.

1 Introduction

If x_n represents the population of an isolated species after n generations, let us suppose this variable is bounded in the range $0 < x_n < 1$. A simple model that gives account of its evolution is the so-called logistic map [1],

$$x_{n+1} = \mu\, x_n(1 - x_n), \tag{1}$$

where $0 < \mu < 4$ in order to assure $0 < x_n < 1$. The term μx_n controls the *activation or expanding phase*, where μ expresses the *growth rate*. The term $(1 - x_n)$ inhibits the overcrowding, then it is controlling the *inhibition or contracting phase*. The continuous version of this model was originally introduced by Verhulst in the nineteenth century as a counterpart to the Malthusian theories of human overpopulation.

2 The Models

If two species (x_n, y_n) are now living together [2], each evolves following a logistic-type dynamics,

$$\begin{aligned} x_{n+1} &= \mu_x(y_n)\, x_n(1 - x_n), && (2)\\ y_{n+1} &= \mu_y(x_n)\, y_n(1 - y_n). && (3) \end{aligned}$$

The interaction between species causes the growth rate $\mu(z)$ to vary with time, then $\mu(z)$ depends on the population size of the others and on a positive constant λ that measures the strength of the mutual interaction. The simplest choice for this growth rate can be a linear increasing μ_1 or decreasing μ_2 function expanding at the parameter interval where the logistic map shows some activity, that is $\mu \in (1, 4)$. Thus,

$$\begin{aligned} \mu_1(z) &= \lambda\,(3z + 1), && (4)\\ \mu_2(z) &= \lambda\,(-3z + 4). && (5) \end{aligned}$$

Then we have:
(1) The *symbiosis* originates a symmetrical coupling due to the mutual benefit, then $\mu_x = \mu_y = \mu_1$.
(2) The *predator-prey interaction* is based on the benefit/damage relationship established between the predator and prey, respectively, then $\mu_x = \mu_1$ and $\mu_y = \mu_2$.
(3) The *competition* between species causes the contrary symmetrical coupling, then $\mu_x = \mu_y = \mu_2$.

3 Summary and Conclusions

Three discrete two-dimensional logistic systems are proposed to model the basic interactions between pairs of species. These could be considered for future studies as the bricks necessary to built more complex interaction networks among species.

References

[1] R.M. May, "Simple mathematical models with very complicated dynamics," *Nature*, Vol. 261, pp. 459-467, 1976.

[2] R. López-Ruiz and D. Fournier-Prunaret, "Complex behavior in a discrete logistic model for the symbiotic interaction of two species," *Mathematical Biosciences and Engineering*, Vol. 1 (2), June 2004.